The Space Elevator Feasibility Condition

Ben Shelef, the Spaceward Foundation

Abstract

This paper ties together parameters pertaining to tether specific strength and to power system mass density to arrive at an inequality that determines whether a space elevator system is viable.

The principle for the feasibility condition (FC) is that a space elevator must be able to lift its own weight fast enough – fast enough to grow by bootstrapping, fast enough to replace aging material, and fast enough to have a significant margin for commercial cargo beyond these housekeeping tasks. The FC therefore sets a 3 dimensional design space comprised of {tether material specific strength, power system specific power, system time constant}.

After developing the FC, real life limitations on specific power and specific strength are plugged in, and the resultant viable design space is examined. Finally, a design architecture that satisfies the Feasibility Condition is briefly introduced.

1 Motivation

As is well known, there is no hard minimum requirement on the specific strength of a space elevator tether. The lower the specific strength, the higher the tether taper ratio, and the heavier the tether gets for a given climber mass. Previous work (Edwards, [4]) cites a UTS of 130 GPa for CNTs, and a density of 1.3 g/cc, for a specific strength of 100 GPa-cc/g, and a taper ratio of 1:2. Edwards uses this number as a starting working point, with the implication that if CNT tethers will not reach this specific strength, the effect will be an increase in the taper ratio and in the total mass of the space elevator tether, but this will not be a fundamental problem or show-stopper for the construction of the space elevator.

In a similar fashion, there is no absolute requirement on the performance of the power system of the space elevator. Higher specific power allow the climber to move faster and clear the bottom of the tether sooner, increasing the launch rate and mass throughput of the system. In [4] Edwards cites a 2 MWatt power system weighing 5 tons (0.4 kWatt/kg), able to launch a climber about once a week. Again, the implication is that if this specific power cannot be reached, the only penalty will be a reduction in the possible payload mass throughput, but this will not affect the feasibility of the space elevator.

In previous discussions of space elevator design, tether strength and power systems are treated as mostly independent domains – railroad tracks and train engines, to use a familiar analogy.

There is, however, another assumption in the space elevator architecture that ties these two domain together. It is accepted that a space elevator is too heavy to launch directly, and so the only way to construct a viable-sized space elevator is to launch a smaller seed space elevator, and use its lifting capacity to bootstrap to a much larger space elevator. In addition, the tether material will have a certain expected lifetime in operation, and so the entire tether mass has to be replaced at a certain minimum rate, using the space elevator itself to perform the task.

These "housekeeping" lift chores create the link between the tether system and the power system, since a lower-performance tether requires more housekeeping work, and thus levies a throughput requirement on the power system, thereby denying us the option to simultaneously have both an arbitrarily weak tether and an arbitrarily weak power system. If the housekeeping chores cannot be kept, then not just the performance, but indeed the feasibility of the space elevator is cast into question.

2 Feasibility Condition

To lay out the rationale for the FC, we need to define some basic design parameters for the space elevator.

Since the space elevator is linearly scalable, the discussion below is independent of the size of the system. We therefore normalize all mass parameters by the maximum mass that is allowed to hang from the bottom of the tether (m_{max}) and refer to this unit as a *Standard Mass Unit* (SMU). Thus for example a 20-ton space elevator is one that can support a 20 ton load at ground level, and if its tether weighs 6000 tons, then we say the tether weighs 300 SMUs. The ratio of tether mass to lifting capacity is labeled *Tether Mass Ratio* (TMR) – 300 in this case.

The *Tether Specific Loading* (TSL) is similar to the tether material specific strength, but takes into account parasitic mass (such as cross-weaves) and the margin of safety, so is the effective tether specific strength used in the design. For example, the ribbon material may achieve 40 MYuri, but the TSL used in the design is only 30 MYuri. Given a specific TSL, and using the constant-stress space elevator tether formula, it is possible to calculate the taper ratio, total mass, and thus the TMR.

Shifting the attention to the power system side of the design, the *Payload Mass Throughput* (PMT) of the space elevator is defined as the amount of mass it can move per unit time. The normalized unit for PMT is the *Standard Throughput Unit* (STU), defined as one SMU per year. The PMT is highly dependent on the specific power (SP) of the power system, measured in kWatt/kg, but this dependency is more complex than the one between the TSL and TSR.

Simplistically, the mass of the climber is divided into payload and power system (neglecting structure and other mass overhead), and the *payload mass ratio* (PMR) is defined as the mass of the payload divided by the total climber mass. Optimal PMT depends on the PMR (a higher PMR means the climbers carry more payload, but move slower)

Optimal PMT also depends on the number of concurrent climbers on the tether. Since each climber further up the tether weighs much less than a climber that is just taking off, it pays to have multiple climbers, each being lighter than 1 SMU. (Thus for example a 20-ton space elevator can operate 5–6 concurrent 15 ton climbers). In this respect, optimal PMT is achieved by having a continuous stream of infinitesimal climbers, but the optimization process is capped much earlier by the interactions between the power system and the Earth's rotation, limiting climbers to a once-per-day schedule.

A more detailed treatment of PMT optimization is presented in [12]. We are using the case limited by a once-daily cycle, since further (unpublished) work in respect to direct-solar power systems is making this to be the likely case to be used.

It is thus possible to derive the TMR from the TSL (through the constant-stress formula) and the PMT from the SP (through optimization and some operational considerations). The FC will create a dependency between the TMR and the PMT.

The Space Elevator Feasibility Condition (FC) is built around the concept of the characteristic time constant (CTC) – the time it takes the system to lift a payload mass equal to the mass of its tether. With all of the above parameters defined, the CTC is simply the ratio of the tether mass and the payload mass throughput of the system: CTC = TMR/PMT. The reciprocal, 1/CTC, is therefore a measure of the Normalized Throughput (NT) of a space elevator system. A space elevator with a CTC of 2 years can lift half of itself into orbit each year, so has an NT of 0.5 yr^{-1}.

The CTC can now be compared with several time constants that are required under some operational assumptions.

We call the periods where the space elevator bootstraps, either during initial construction or during recovery from a break, “growth periods”, to be contrasted with “normal operations”. (Bootstrapping is the process of growing the space elevator’s tether using the existing tether as the transport mechanism)

During growth periods, the space elevator relies on a certain growth rate, or time-to-double (TD). It is generally accepted that the size of the rocket-launched seed elevator is about 5% - 10% of the size of the first viable space elevator. This means that the space elevator has to double in size 3–4 times between its seed state and its operational state. We would argue that for a commercial transportation infrastructure project, 8–12 years is a reasonable limit to the amount of time it can be under construction before carrying payloads, and so TD of 2–3 years is reasonable. Civil infrastructure projects (E.g. Golden Gate Bridge, Transcontinental Railroad, British Channel Tunnel) all took a similar number of (or fewer) years to complete.)

Since we want to hold a spare tether spool in orbit to enable reasonable recovery from a tether break event, we need to count on this additional mass being launched as well. We’ll denote the fraction weight of the spare-in-orbit as FS. If, for example, FS = 25%, then it will take 2 doublings (and thus 2*TD years) to fully recover from the a broken tether.

There is no absolute requirement on TD and FS, but we’ll proceed right now with the notional values TD=1.5 and FS=25%.

During normal operations, the tether material will have some degradation rate in space as a result of cosmic radiation, micro orbital debris, chemical interactions, thermal cycling, and simple mechanical wear and tear. These factors will result in an allowed material lifetime in service, denoted TL. If any specific portion of the tether is prone to a shorter lifespan, we’ll amortize that into the overall TL figure according to the size of that segment. TL is not currently known and depends partially on specific tether properties, but it is reasonable to assume TL >> TD given the values above.

To support growth, the space elevator must lift (1+FS)/TD of its mass per year. In a similar manner, to replace aging material, the space elevator must lift 1/TL of its mass per year. These are the housekeeping lift requirements introduced above.

Since during growth periods the housekeeping lift requirements are much higher than during periods of normal operations, it is enough to require that the space elevator will exactly support the housekeeping operations during growth periods, and use the additional capacity during normal operations for lifting payload. The sum (1+FS)/TD + 1/TL is therefore the minimal *required* normalized throughput of the space elevator, in units of ribbon-masses-per-year (yr^{-1}).

The Feasibility Condition can thus be phrased as follows:

$$\frac{PMT}{TMR} = \frac{1}{CTC_{pos}} > NT_{req} = \frac{1+FS}{TD} + \frac{1}{TL}$$

Obviously, if the feasibility condition can be met, and since during normal operations only the second term (1/TL) is present, the throughput represented by the first term (TMR·(1+FS)/TD) is all available for payload transportation.

3 CNTs and Power Beaming System Performance

In this section we’ll try to estimate the values of PMT and TMR achievable in the foreseeable future. As such estimates go, these will only establish a very rough range, since the technologies are complex and we are trying to look rather far into the future. The goal here is to merely estimate these values to an order of magnitude, so we can determine whether the FC shows the space elevator as “easily feasible”, “clearly infeasible”, or somewhere in between.

3.1 Tether material

Based on a gradual convergence of experimental and theoretical results, the specific strength of raw CNTs [7] will not exceed 50 MYuri [9],[10],[11] as compared to previous estimates of 100 MYuri[4].

In particular, recent work by Yakobson et. al. [11] investigates ab-initio simulations of CNTs under strain at various temperatures, with various symmetries, and at different time spans.

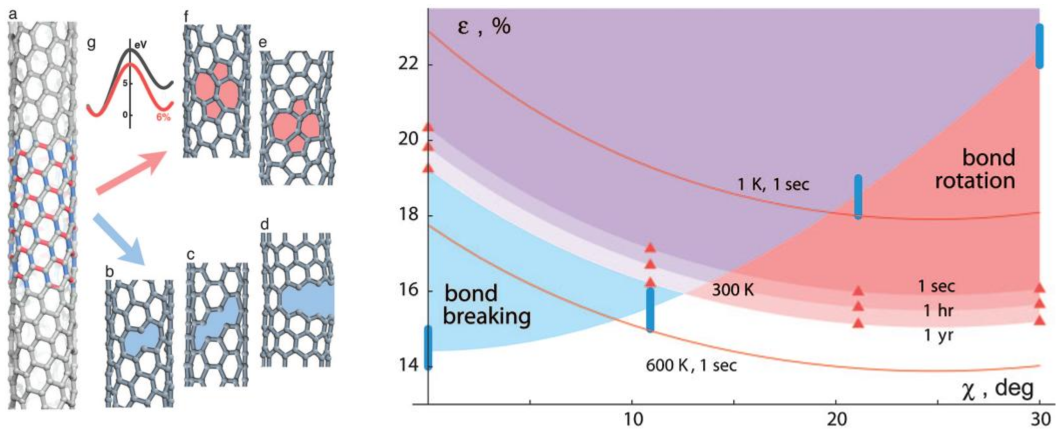

Figure 1: Brittle and Ductile failure of CNTs (Courtesy Boris Yakobson, Rice university)

This simulations agree with experimental results such as reported by Ruoff et al. in [9]

Using 45 MYuri CNTs that are long enough to spin into a tether, a well spun tether (A task that has not been achieved yet) will achieve 90% strength (40 MYuri), and with a 33% safety margin, we can load the tether at a TSL of 25–30 MYuri.

The weight of various redundancy structures can be shown to be only a few percent of the total tether mass, so will not affect this result significantly.

3.2 Power systems

The analysis of the power system is more complex than that of the tether. The specific power of the power system is a function of the photovoltaic receiver, the electric motors, the power electronics, and any required heat-rejection systems. If one of these components has a significantly low specific power, it will be the heaviest component of the system and become the limiting factor.

Ultra lightweight PV receivers (Figure 2) can reach very high specific powers, with 4.3 kWatt/kg already demonstrated [6], together with large deployable structure [7]. The 400 m^2 panel weighs only 32 kg, including the booms and deployment mechanism. (It stows in the little suitcase at the center – note the people in the back for scale). The panel can provide 50 kWatt of electric power under A0 illumination, or 1.5 kWatt/kg. This type of thin-film panel currently converts sunlight at slightly under 10%. When used with monochromatic light, and at more than one sun intensity, performance will of course go up. Looking forward, it is conceivable that PV technology can eventually deliver 5 kWatt/kg under these conditions. It is important to note that the mechanical structure shown was designed for zero g, and a space elevator receiver will have a higher structural mass.

The best electric motors today achieve a specific power of just under 1.0 kWatt/kg, though there is no theoretical barrier preventing this number from improving. Specifically, CNT-based conductors can provide a large leap in performance in that respect, replacing copper in motor windings. Even today's CNT conductors rival copper in specific conductivity. The lightest electrical motors, however, are not the most efficient ones, and since heat rejection carries its own mass penalty, we must have motors that have both high specific power and ultra high efficiency.

Figure 2: Thin Film PV Panel for in-space use

Heat is generated in the system in two primary locations. On the PV receiver itself, and in the drive system.

Heat generated at the receiver is already spread out over a large surface, and is radiated from where it is generated. Therefore the efficiency of the receiver and its allowed operating temperature jointly determine how much laser power can be placed on it. Given the specific powers quoted above even at one solar illumination, power rejection at the panel level will not be an issue.

Heat generated at the motor and power electronics has to be conducted from them to radiators. The amount of heat is of course only a few percent of the output of the motor, but today's heat rejection systems are far too bulky even for this task, since their mass is dictated by fluid-based heat distribution plumbing and by the specific heat conductivity of the fin material. The heat rejection system of the ISS, for example, operates at less than 0.01 kWatt/kg. If the power system is 95% efficiency, than this value is equivalent to 0.2 kWatt/kg when considering drive power.

There is no fundamental reason, however, why this is so. The fin area itself can be made extremely lightweight, and the heat rejection temperature of an electric motor is higher than that of the ISS. Availability of lightweight highly conductive materials to replace coolant distribution loops and copper will reduce the weight of heat rejection systems by a large factor. Since the specific heat conductivity of CNTs is so much higher than that of aluminum (better than 100x) it is clear that great strides can be made in this field.

An additional factor that helps with power dissipation is that high power operations occur during the first night, when the radiators are more efficient. Finally, it is also possible to transfer heat to the tether as it passes by the climber. This is especially attractive for heat generated within the hubs of the rollers.

It can be surmised that heat rejection systems are the most likely limiting factor for the specific power of the power system, but that the current limit can be greatly improved upon. We'll proceed with the assumption that the power system can be built to perform at between 1 and 1.5 kWatt/kg

3.3 Normalized Throughput

The required values for the normalized throughput are a function of TD, TL, and FS as discussed above. Keeping the assumption of FS=0.25, Table 1 shows the values of NT=1/CTC as a function of TD and TL.

TL is a function of the tether material and space environment. TD is business-derived. Even though TL is not known at this point and TD is a business-driven quantity , the likely range of values for NT is rather narrow – between 0.5 and 1.5.

FS=25%		TL (yrs)		
		4	6	10
TD (yrs)	1	1.50	1.42	1.35
	1.5	1.08	1.00	0.93
	2	0.88	0.79	0.73
	3	0.67	0.58	0.52

Table 1: Required normalized throughput values
$NT_{req} = 1/CTC_{req} = (1+FS)/TD + 1/TL$

4 Numerical Values

We can now take the estimates of section 3 and see how they fit into the FC.

For example:
A space elevator constructed with a 30 MYuri tether and 3.5 kWatt/kg motors, can lift its own mass 1.7 times a year.

	TMR Tether mass ratio	**A/A_0** Taper ratio	**TSL** Tether specific loading (MYuri[i])	SP 0.5 / PMT 100	SP 0.7 / PMT 135	SP 1.0 / PMT 170	SP 1.5 / PMT 200	SP 2.5 / PMT 230	SP 3.5 / PMT 250
optimistic	**50**	2.6	50	**2.0**	**2.7**	**3.4**	**4.0**	**4.6**	**5.0**
	77	3.4	40	**1.3**	**1.8**	**2.2**	**2.6**	**3.0**	**3.2**
CNT	**144**	5.0	30	**.69**	**.94**	**1.2**	**1.4**	**1.6**	**1.7**
CNT	**228**	7.0	25	**.44**	**.59**	**.75**	**.88**	**1.0**	**1.1**
	433	11.3	20	**.23**	**.31**	**.39**	**.46**	**.53**	**.58**
pessimistic	**739**	17.3	17	**.14**	**.18**	**.23**	**.27**	**.31**	**.34**
				pessimistic		Thin-film receiver + motors	Thin-film receiver + motors		optimistic

SP Specific power (kWatt/kg); **PMT** Optimized payload throughput (STU)

Blue values are optimistic, Red values are pessimistic, gray values are probable. As expected, stronger tethers and lighter power systems influence the normalized throughput in a similar manner and so are interchangeable.

***Table 2**:* Tether vs. power driven constraints and their effect on the possible normalized throughput:
$NT_{pos} = 1/CTC_{pos} = PMT/TMR$

Given realistic technology values (marked in gray) we can see that the feasible normalized throughput, 1/CTC, is in the range of 0.75 – 1.4.

5 Conclusion

The Space Elevator Feasibility Condition is a sufficient but not a necessary condition for the viability of the space elevator as a practical transport system.

However, it is “strongly sufficient” in that it is able to define and capture the principal requirements on the strength of the tether and the specific power of the power system. Without it, we don’t really have a sufficient condition, since a space elevator can be built out of any tether material (since at worst, the tether will simply weigh more) and using any power system (since at worst, the climbers will simply move more slowly.)

The above analysis shows that the possible values of CTC match the required values very tightly, under the assumptions made. This means the feasibility condition can be satisfied, but not without difficulty. The connection between the tether and power systems means that the power system, and specifically the heat rejection system, is as important a design challenge as the design of the tether.

Additionally, the Feasibility Condition alerts us to other aspects of the design of the space elevator. For example, the handling of bi-directional traffic and the disposition of climbers after they have carried their cargo. If the climbers had to come down the same tether, then this affects the throughput of the system, and thus factors into the FC. If the climbers use a second tether as a “down” elevator, then this tether increases the mass of the system since it also has to be maintained. If the climbers are cast off after each use, then we need to bring into account the cost of disposable climbers, which is significant.

The last observation ties the Feasibility Condition to a future financial study of the space elevator. We often portray the space elevator as a “free” system. The tether is an invariant, the climbers reusable, and the cost of electricity to power the laser can quickly be calculated to be negligible (especially if solar light is used for most of the trip).

However, for a full financial model we need to factor in the cost of tether replacement, and perform financial trade-offs such as choosing between reducing throughput, using more tether material (as in a “down” tether), or disposing of the climbers. This will result in a “Space Elevator Financial Viability Condition”, which will impose a tighter bound on its real feasibility. While the values of the technology parameters are still not known, the mathematical model can be constructed today.

6 References

[1] Artsutanov, Y. “Into the Cosmos by Electric Rocket”, *Komsomolskaya Pravda,* 31 July 1960. (The contents are described in English by Lvov in *Science,* **158**, 946-947, 1967.)

[2] Artsutanov, Y. “Into the Cosmos without Rockets”, *Znanije-Sila* **7**, 25, 1969.

[3] Pearson, J. “The Orbital Tower: A Spacecraft Launcher Using the Earth's Rotational Energy”, *Acta Astronautica* **2**, 785-799, 1975.

[4] Edwards, B. C.; Westling, E. A. “The Space Elevator: A Revolutionary Earth-to-Space Transportation System”, ISBN 0972604502, published by the authors, January 2003s

[5] Mason, L. S. “A Solar Dynamic Power Option for Space Solar Power”, Technical Memorandum NASA/TM—1999-209380 SAE 99–01–2601, 1999

[6] Wyrsch, N. & 8 co-authors (2006) “Ultra-Light Amorphous Silicon Cell for Space Applications," Presented at 4th World Conference and Exhibition on Photovoltaic Solar Energy Conversion, March 2006, Waikoloa, Hawaii

[7] G Laue, D Case, & J Moore, 10-13 July 2005, “Fabrication and Deployment Testing of Solar Sail Quadrants for a 20-meter Solar Sail Ground Test Demonstration”, *AIAA2005-3930*, 41st AIAA/ASME/SAE/ASEE Joint Propulsion Conf. & Exhibit, Tucson, AZ.

[8] S. Iijima "Helical microtubules of graphitic carbon", *Nature* 56, 354 (1991)

[9] Ruoff et al. “Mechanical properties of carbon nanotubes: theoretical predictions and experimental measurements”, *C. R. Physique* 4 [2003]

[10] T. Belytschko et al “Atomistic Simulations of Nanotube Fracture”, *PHYSICAL REVIEW B* [2002]

[11] Dumitrica, T; Hua, M; Yakobson, BI "Symmetry-, time-, and temperature-dependent strength of carbon nanotubes " *Proceedings of the National Academy of Sciences of the United States of America*, 103 2006/APR: 6105-6109

[12] B. Shelef "Space Elevator Power System Analysis and Optimization", *The Spaceward Foundation*, 2008

[13] B. Shelef "Space Elevator Calculation Scrapbook", *The Spaceward Foundation*, 2008

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[i] A Yuri is the SI derived unit for specific strength. 1 MYuri = 1 N/Tex = 1 GPa-cc/g